IMO

SOF INTERNATIONAL MATHEMATICS OLYMPIAD 2023-24

CLASS 11

QUESTION PAPER SET A

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DO NOT OPEN THIS BOOKLET UNTIL ASKED TO DO SO

Total Questions: 50 | Time: 1 hr.

Guidelines for the Candidate

1. You will get additional ten minutes to fill up information about yourself on the OMR Sheet, before the start of the exam.
2. Write your **Name, School Code, Class, Section, Roll No.** and **Mobile Number** clearly on the **OMR Sheet** and do not forget to sign it. We will share your marks / result and other information related to SOF exams on your mobile number.
3. The Question Paper comprises four sections :
   Section - 1 : **Logical Reasoning** (15 Questions)
   Section - 2 : **Mathematical Reasoning** (20 Questions) or **Applied Mathematics** (20 Questions)
   Section - 3 : **Everyday Mathematics** (10 Questions)
   Section - 4 : **Achievers Section** (5 Questions)
4. **Section-1, 3 and 4 are compulsory for all**. In Section-2 opt for Mathematical Reasoning OR Applied Mathematics and mark the same on the OMR Sheet.
   Each question in Achievers Section carries 3 marks, whereas all other questions carry one mark each.
5. All questions are compulsory. There is no negative marking. Use of calculator is not permitted.
6. There is only ONE correct answer. Choose only ONE option for an answer.
7. To mark your choice of answers by darkening the circles on the OMR Sheet, use **HB Pencil** or **Blue / Black ball point pen** only. E.g.
   Q.16: Rahul bought 4 kg 90 g of apples, 2 kg 60 g of grapes and 5 kg 300 g of mangoes. The total weight of all the fruits he bought is_____.
   A. 11.450 kg   B. 11.000 kg   C. 11.350 kg   D. 11.250 kg
   As the correct answer is option A, you must darken the circle corresponding to option A on the OMR Sheet.

   16. ● Ⓑ Ⓒ Ⓓ

8. Rough work should be done in the blank space provided in the booklet.
9. Return the OMR Sheet to the invigilator at the end of the exam.
10. Please fill in your personal details in the space provided before attempting the paper.

Name:..

SOF Olympiad Roll No.:... Contact No.:...
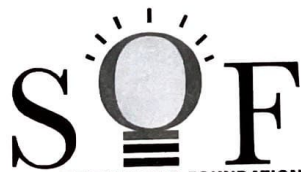
S O F
SCIENCE OLYMPIAD FOUNDATION
Inspiring Young Minds Through Knowledge Olympiads

1. In the given Venn diagram, rectangle represents males, triangle represents people working in armed forces, circle represents doctors and square represents people working in rural areas. Which of the following number represents females who are working in armed forces as a doctor in urban areas?
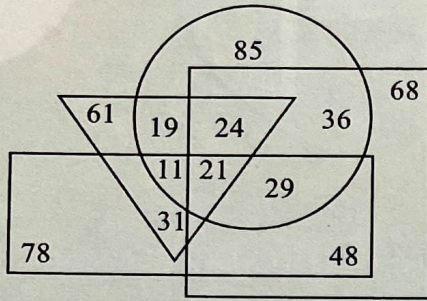
A. 24
B. 19
C. 21
D. 36

2. If the first and the last digit of each of the given numbers are interchanged and two is added to the middle digit, then what is the sum of the digits of the second highest number formed?

573 926 465 757 846

A. 21
B. 19
C. 18
D. 20

3. There is a set of four figures P, Q, R and S showing a sequence of folding of a piece of paper. Figure S shows the manner in which the folded paper has been cut. Select a figure from the options which would most closely resemble the unfolded form of figure S.


P   Q   R   S

A.

B.

C.

D.
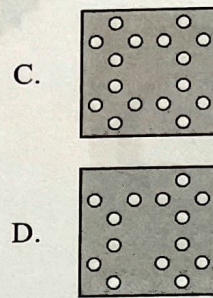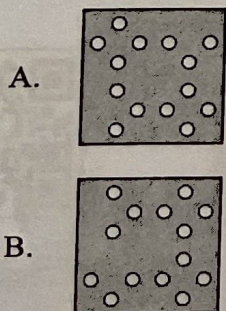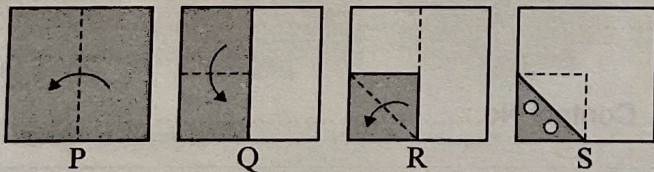
4. Eight persons P, M, R, T, Q, U, V and W are sitting in front of one another in two rows. Each row has 4 persons. P is between U and V and facing North. Q, who is to the immediate left of M is facing W. R is between T and M. W is to the immediate right of V. Who is sitting in front of R?
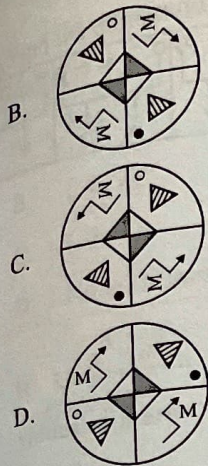
A. U
B. Q
C. V
D. P

5. Select a figure from the options which is exactly embedded in the given figure as one of its parts.

A.

B.

C.

D.
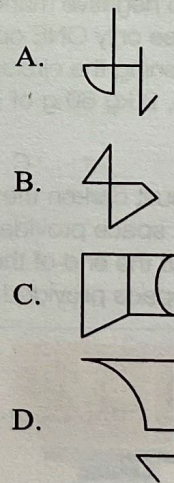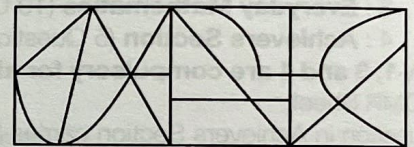
6. Select the correct water image of the given figure.

A.

B.

C.

D.
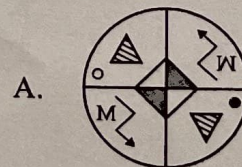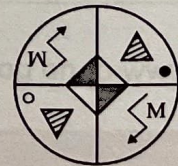
7. Study the given information carefully and answer the following question.

'A + B' means 'A is the son of B';
'A × B' means 'A is the mother of B';
'A − B' means 'A is the brother of B';
'A ÷ B' means 'A is the daughter of B'.

What does the expression J × L + K − M ÷ N denotes?

A. J is the daughter-in-law of N
B. L is the nephew of M
C. N is the mother of M
D. Both A and B

8. How many such pairs of letters are there in the word COMPLETION each of which has the same number of letters between them in the word as in the English alphabets?

A. One
B. Two
C. Three
D. More than three

9. In a certain code language, 'hope to see you' is coded as 're so na di', 'please come to see the party' is coded as 'fi ge na di ke zo', 'hope to come' is coded as 'di so ge' and 'see you the party' is coded as 're fi zo na'.
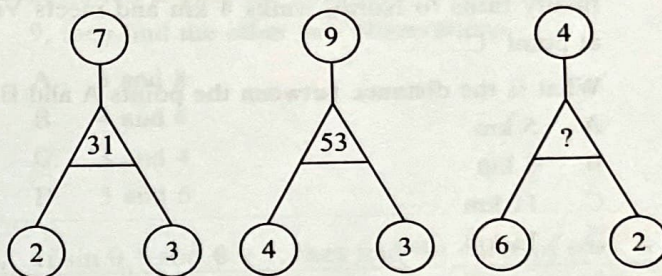
How will 'please' be coded in the given code language?

A. di
B. ke
C. fi
D. na

10. Three different positions of a dice are shown below. Find the number of dots on the face opposite to the face having three dots.
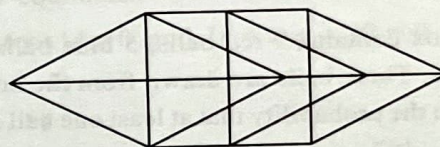

A. Six
B. Two
C. Five
D. One

11. Find the missing number, if same rule is followed in all the three figures.

A. 28
B. 25
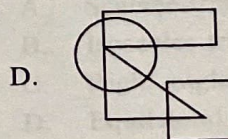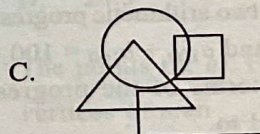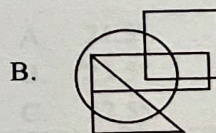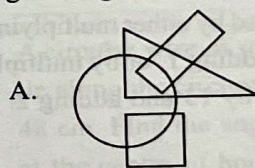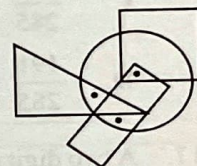C. 21
D. 60

12. Find the number of triangles formed in the given figure.

A. 20
B. 21
C. 22
D. More than 22

13. Which of the following options satisfies the same conditions of placement of the dots as in the given figure?

A.

B.

C.

D.
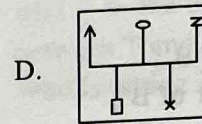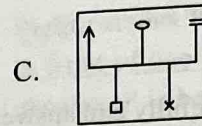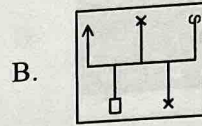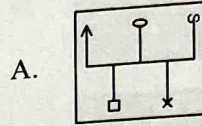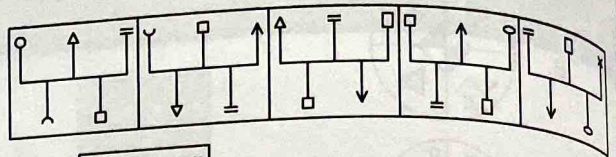
14. Vijay and Ravi starts walking from two different points 'A' and 'B' respectively. Vijay walks 2 km North and turns to the East and walks 3 km and again turns to North and walks 4 km and finally turns to East and walks 5 km to reach point 'C'. Similarly, Ravi walks 2 km North and turns to West and walks 3 km and finally turns to North, walks 4 km and meets Vijay at point 'C'.

What is the distance between the points A and B?

A. 5 km
B. 8 km
C. 11 km
D. 13 km

15. Select a figure from the options which will continue the same series as established by the Problem Figures.

A.
B.
C.
D.

MATHEMATICAL REASONING

16. A box contains 9 red balls, 5 blue balls and 6 green balls. Three balls are drawn from the box at random. Find the probability that at least one ball will be green.

A. $\dfrac{187}{295}$

B. $\dfrac{194}{285}$

C. $\dfrac{94}{285}$

D. $\dfrac{191}{285}$

17. A two digit number is obtained by either multiplying the sum of the digits by 8 and adding 1, or by multiplying the difference of the digits by 13 and adding 2. Find the number.

A. 41
B. 42
C. 43
D. None of these

18. Let $a_1, a_2, ...$ and $b_1, b_2, ...$ be two arithmetic progressions such that $a_1 = 25$, $b_1 = 75$ and $a_{100} + b_{100} = 100$, then the sum of first hundred terms of the progression $a_1 + b_1, a_2 + b_2, ...$ is equal to

A. 1000
B. 100000
C. 10000
D. 24000

19. Let $U = R$. If $A = \{x \in R : 0 < x < 2\}$, $B = \{x \in R : 1 < x \le 3\}$, then which of the following is incorrect?

A. $A' = \{x \in R : x \le 0$ or $x \ge 2\}$
B. $B' = \{x \in R : x \le 1$ or $x > 3\}$
C. $A \cup B = \{x \in R : 0 \le x \le 3\}$
D. $A \cap B = \{x \in R : 1 < x < 2\}$

20. In a right $\triangle ABC$ right-angled at C, if D is the midpoint of BC, then $(AD^2 - AC^2) =$

A. BC^2
B. $2BC^2$
C. $\dfrac{BC^2}{4}$
D. None of these

21. The value of $^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is

A. $^{55}C_4$
B. $^{55}C_3$
C. $^{56}C_3$
D. $^{56}C_4$

22. The radius of a spherical balloon increases from 8 cm to 12 cm. The ratio of the surface areas of the balloon in two cases is

A. $2 : 3$
B. $3 : 2$
C. $8 : 27$
D. $4 : 9$

23. Find the domain for which the functions $f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ are equal.

A. $\left\{-2, \dfrac{-1}{2}\right\}$

B. $\{-2, 2\}$

C. $\{-1, 1\}$

D. None of these

24. If $\underset{x \to a}{Lt} \dfrac{x^9 - a^9}{x - a} = \underset{x \to 5}{Lt} (x + 4)$, then all possible real values of a are

A. 2, 3

B. -2, 2

C. -1, 1

D. -3, 3

25. If α and β are the zeroes of the polynomial $f(x) = x^2 - 5x + k$ such that $\alpha - \beta = 1$, then

A. $\alpha\beta = 6$

B. $\alpha^2 + \beta^2 = 13$

C. $k = 6$

D. All of these

26. The number of ways in which an examiner can assign 30 marks to 8 questions giving not less than 2 marks to any question is

A. $^{19}C_7$

B. $^{20}C_7$

C. $^{21}C_7$

D. $^{22}C_7$

27. The value of $\dfrac{i^6 + i^7 + i^8 + i^9}{-1 + i} =$

A. $2i$

B. 0

C. $1 + i$

D. 1

28. Find the roots of the quadratic equation $x^2 + 4\sqrt{2}x + 6 = 0$.

A. $-\sqrt{2}, -3\sqrt{2}$

B. $\sqrt{3}, 2\sqrt{3}$

C. $\sqrt{2}, -3\sqrt{2}$

D. $\sqrt{2}, -5\sqrt{2}$

29. The centres of the circles $x^2 + y^2 - 6x - 8y - 7 = 0$ and $x^2 + y^2 - 4x - 10y - 3 = 0$ are the ends of the diameter of the circle _____.

A. $x^2 + y^2 - 5x - 9y + 26 = 0$

B. $x^2 + y^2 + 5x - 9y + 14 = 0$

C. $x^2 + y^2 + 5x - y - 14 = 0$

D. $x^2 + y^2 + 5x + y + 14 = 0$

30. The mean of 5 observations is 6 and the standard deviation is 2. If the three observations are 5, 7 and 9, then find the other two observations.

A. 3 and 8

B. 4 and 6

C. 3 and 4

D. 3 and 6

31. If $\sin\theta + \sin^2\theta = 1$, then find the value of $\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta + 2\cos^4\theta + 2\cos^2\theta - 2$.

A. 1

B. 2

C. 0

D. -1

32. A point equidistant from the lines $x + \sqrt{3}y + 4 = 0$, $\sqrt{13}x + 6y + 14 = 0$ and $7x + 24y - 50 = 0$ is

A. $(1, -1)$

B. $(1, 1)$

C. $(0, 0)$

D. $(0, 1)$

33. A bicycle wheel makes 5000 revolutions in moving 11 km. Find the diameter of the wheel.

A. 64 cm

B. 70 cm

C. 68 cm

D. 72 cm

34. A circular wire of radius 3 cm is cut and bent so as to lie along the circumference of a hoop whose radius is 48 cm. Find the angle in degrees which is subtended at the centre of hoop.

A. $21.5°$

B. $23.5°$

C. $22.5°$

D. $24.5°$

35. The points (a, a), $(-a, -a)$ and $(-\sqrt{3}a, \sqrt{3}a)$ are the vertices of a/an _____.

A. Scalene triangle

B. Isosceles triangle

C. Right-angled triangle

D. Equilateral triangle

16. The roots of the equation $\dfrac{x+3}{x+2} = \dfrac{3x-7}{2x-3}$ are

A. 5, 1

B. −5, −1

C. −3, −2

D. 5, −1

17. Solve for x : $\log_x 2401 = -4$

A. 7

B. $\dfrac{1}{7}$

C. 14

D. $\dfrac{-1}{14}$

18. If α and β are the zeroes of the polynomial $f(x) = x^2 + px + q$, then a polynomial having $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$ as its zeroes, is

A. $x^2 + qx + p$

B. $x^2 - px + q$

C. $qx^2 + px + 1$

D. $px^2 + qx + 1$

19. Let X, Y, Z be the subsets of U, where $n(U) = 35$, $n(X) = 15$, $n(Y) = 22$, $n(Z) = 14$, $n(X \cap Y) = 11$, $n(Y \cap Z) = 8$, $n(X \cap Z) = 5$, $n(X \cap Y \cap Z) = 3$, then $n(X \cup Y \cup Z)'$ equals

A. 35

B. 30

C. 26

D. 5

20. If kurtosis of a normal distribution is 'a' and skewness is 'b', then $a - b$ is equal to

A. 0

B. 2

C. 3

D. 1

21. The equation of the line passing through the point $(-3, 5)$ and perpendicular to the line through the points $(2, 5)$ and $(-3, 6)$ is

A. $3x + y - 15 = 0$

B. $5x - y + 20 = 0$

C. $2x + y - 7 = 0$

D. $5x + 5y - 11 = 0$

22. A man borrowed a sum of money and agrees to pay off by paying ₹ 3700 at the end of the first year and ₹ 4935 at the end of the second year. If the rate of compound interest is 5% per annum, then find the sum borrowed.

A. ₹ 7500

B. ₹ 8000

C. ₹ 8200

D. ₹ 8400

23. $\dfrac{1}{1+\sin A} + \dfrac{1}{1-\sin A} =$

A. $2 \sec^2 A$

B. $\sec^2 A$

C. $\cos^2 A$

D. $\csc^2 A$

24. From the following table, calculate the Karl Pearson's coefficient of correlation.

X	Y
6	9
3	11
10	?
4	8
12	7

Arithmetic means of X and Y series are 7 and 8 respectively.

A. −0.619

B. 0.712

C. −0.779

D. 0.612

25. 20 persons are invited for a party. In how many different ways can they and the host be seated at a circular table, if the two particular persons are to be seated on either side of the host?

A. 20!

B. $2 \times 18!$

C. 18!

D. None of these

26. Find the mean of the following frequency distribution.

Class interval	Frequency
10 – 30	5
30 – 50	8
50 – 70	12
70 – 90	20
90 – 110	3
110 – 130	2

A. 64.7
B. 66.5
C. 65.6
D. 67.6

27. Convert the binary number 1101101110 to the decimal number.

A. 881
B. 880
C. 879
D. 878

28. The radius of a circle is 20 cm. It is divided into four parts of equal areas by drawing three concentric circles inside it. Then, the radius of the largest of three concentric circles drawn is _____.

(Take π = 3.14)

A. $10\sqrt{5}$ cm
B. $10\sqrt{3}$ cm
C. 10 cm
D. $10\sqrt{2}$ cm

29. A PG for girls has following appliances, when energy is supplied at 200V and costs ₹ 12 per kWh (or per unit).

(i) 50 bulbs of 100W each, working 8 hours a day.

(ii) 25 fans of 150W each and working 16 hours a day.

(iii) 2 TV sets of 200W each, working 2 hours a day.

(iv) 2 electric motors of 1.5 HP each and working 4 hours a day. (Use: 1 HP = 746 W)

Calculate the monthly electricity bill of the PG.

A. ₹ 37654
B. ₹ 38594
C. ₹ 38792
D. None of these

30. In a triangle DEF, O is the centre of the incircle ABC. $\angle DEF$ = 60°, $\angle DFE$ = 75°. Find $\angle AOB$.
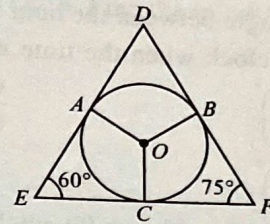
A. 75°
B. 45°
C. 135°
D. None of these

31. Determine the values of a and b so that the following system of linear equations have infinitely many solutions :

$$(2a - 1) x + 3y - 5 = 0$$
$$3x + (b - 1) y - 2 = 0$$

A. $a = \dfrac{17}{4}, b = \dfrac{9}{5}$

B. $a = \dfrac{15}{4}, b = \dfrac{11}{5}$

C. $a = \dfrac{17}{4}, b = \dfrac{13}{5}$

D. $a = \dfrac{17}{4}, b = \dfrac{11}{5}$

32. Squares each of side 5 cm are cut off from the four corners of a sheet of tin measuring 40 cm by 30 cm. The remaining portion of the tin sheet is made into an open box by folding up the flaps. Find the capacity of the box.

A. 5625 cm^3
B. 4265 cm^3
C. 3000 cm^3
D. 2625 cm^3

33. If $P(B) = \dfrac{3}{5}$, $P(A|B) = \dfrac{1}{2}$ and $P(A \cup B) = \dfrac{4}{5}$, then

$$P(A \cup B)' + P(A' \cup B) =$$

A. $\dfrac{1}{5}$

B. $\dfrac{4}{5}$

C. $\dfrac{1}{2}$

D. 1

34. Find the angle between the hour hand and the minute hand of a clock when the time is 5 : 35.

A. $\left(\dfrac{85}{2}\right)^\circ$

B. $\left(\dfrac{75}{2}\right)^\circ$

C. $\left(\dfrac{76}{5}\right)^\circ$

D. $\left(\dfrac{77}{5}\right)^\circ$

35. The sum of four numbers is 64. If we add 3 to first number, subtract 3 from the second, third is multiplied by 3 and fourth be divided by 3, then the all numbers assume the same value. The difference between the largest and smallest of the original numbers is

A. 36

B. 32

C. 27

D. 25

EVERYDAY MATHEMATICS

36. To fill 12 vacancies, there are 25 candidates, of which 5 are from city X. If 3 of the vacancies are reserved for the candidates from city X, while the rest are open to all, then find the number of ways in which selection can be made.

A. 4986200

B. 4974200

C. 4874300

D. 4875200

37. A and B together can do a job in 7 days. If A alone can do the same job in 15 days, then how many days B alone take to complete the job?

A. 8 days

B. 9 days

C. 11 days

D. None of these

38. The milk and water in vessels A and B are in the ratio 5 : 3 and 3 : 4 respectively. In what ratio, the liquids in both vessels be mixed to get a new mixture in vessel 'C' consisting fifty percent milk and fifty percent water?

A. 7 : 4

B. 5 : 2

C. 4 : 7

D. 2 : 5

9. The speed of a boat in still water is 12 km/h, if it goes 24 km downstream and 18 km upstream in the equal time, the speed of boat in downstream and upstream are respectively

A. 96 km/h and 72 km/h

B. 48 km/h and 36 km/h

C. 24 km/h and 18 km/h

D. $\dfrac{96}{7}$ km/h and $\dfrac{72}{7}$ km/h

40. An aeroplane flying with uniform speed horizontally 2 kilometer above the ground is observed at an elevation of $60°$. After 10 s, if the elevation from the same point is observed to be $45°$, then the distance travelled by the aeroplane is

A. $\dfrac{2(\sqrt{3}-1)}{\sqrt{3}}$ km

B. $2(\sqrt{3}+1)$ km

C. $\dfrac{\sqrt{3}-1}{\sqrt{3}}$ km

D. None of these

41. A gentleman buys every year Bank's cash certificates of value exceeding the last year's purchase by ₹ 250. After 20 years, he finds the total value of purchased certificates by him is ₹ 72500. Find the value of the certificates purchased by him in the 13th year.

A. ₹ 4280

B. ₹ 4250

C. ₹ 4300

D. ₹ 4520

42. Ranjeet makes a deposit of ₹ 100000 in Punjab National Bank for a period of 2 years. If the rate of interest is 12% p.a. compounded half yearly, then what will be the maturity value (approx.) of the money deposited by him?

A. ₹ 122247.89

B. ₹ 122436.89

C. ₹ 126247.70

D. ₹ 122436.79

43. A person bought two bicycles for ₹ 1600 and sold the first at 10% profit and second at 20% profit. If he sold the first at 20% profit and second at 10% profit then he would get ₹ 5 more. Find the difference in the cost price of two bicycles.

A. ₹ 60
B. ₹ 75
C. ₹ 50
D. ₹ 80

44. A, B and C enter into a partnership by investing in the ratio of 3 : 2 : 4. After one year, B invests another ₹ 270000 and C, at the end of 2 years, also invests ₹ 270000. At the end of three years, profits are shared in the ratio of 3 : 4 : 5. Find the initial investment of B.

A. ₹ 270000
B. ₹ 360000
C. ₹ 180000
D. ₹ 90000

45. A rectangular lawn 60 m × 40 m has two roads, each 5 m wide running in the middle of it, one parallel to length and the other parallel to breadth. The cost of graveling the roads at 80 paise per sq. m is

A. ₹ 380
B. ₹ 385
C. ₹ 400
D. Data inadequate

ACHIEVERS SECTION

46. Read the given statements carefully and select the correct option.

Statement-I : If the four letter words (need not to be meaningful) are to be formed using the letters of the word "MEDITERRANEAN" such that the first letter is R and the fourth letter is E, then the total number of such words, is 110.

Statement-II : Seven different lecturers are to deliver lectures in seven periods of a class on a particular day. P, Q and R are three of the lecturers. The number of ways in which a routine for the day can be made such that P delivers his lecturers before Q and Q before R, is 210.

A. Both Statement-I and Statement-II are true.
B. Both Statement-I and Statement-II are false.
C. Statement-I is true but Statement-II is false.
D. Statement-I is false but Statement-II is true.

47. Two dice are thrown and the sum of the numbers which come up on the dice is noted. Let us consider the following events associated with this experiment.

A : "the sum is even".
B : "the sum is a multiple of 3".
C : "the sum is less than 4".
D : "the sum is greater than 11".

Which pair of these events is mutually exclusive?

A. A and B
B. B and C
C. C and D
D. A and C

48. Solve the following and select the correct option.

(i) Find the equation of a circle having $(1, -2)$ as its centre and passing through the intersection of the lines $3x + y = 14$ and $2x + 5y = 18$.

(ii) Find the equation of the circle which touches both the axes in the first quadrant and whose radius is a.

	(i)	(ii)
A.	$x^2 + y^2 + 2x - 2y = 0$	$x^2 + y^2 - ax - ay + a = 0$
B.	$x^2 + y^2 - 2x + 4y - 20 = 0$	$x^2 + y^2 - 2ax - 2ay + a^2 = 0$
C.	$2x^2 + y^2 - x + 2y - 10 = 0$	$x^2 + y^2 + 2ax + a^2 = 0$
D.	$2x^2 + 2y^2 - x - 2y - 5 = 0$	$x^2 + y^2 - ax + 2ay + a = 0$

49. Solve the following:

(i) $\lim\limits_{x \to 0} \dfrac{\log(1+ax) - \log(1-bx)}{x} =$

(ii) If $\lim\limits_{x \to 2} f(x)$ may exist, where $f(x) = \begin{cases} 4x - 5, x \le 2 \\ x - k, x > 2 \end{cases}$ then the value of k is _____.

	(i)	(ii)
A.	ab	-1
B.	$a + b$	-1
C.	$a + b$	2
D.	$a - b$	0

50. Which of the following options is incorrect?

A. If the first and the nth terms of a G.P. are a and b respectively and P is the product of the first n terms, then $P^2 = (ab)^n$.

B. The first and last term of an A.P. are a and l respectively. If S be the sum of all the terms of the A.P., then the common difference is $\dfrac{l^2 - a^2}{2S - (l - a)}$.

C. Let a, b, c be in A.P. and $|a| < 1, |b| < 1, |c| < 1$. If $x = 1 + a + a^2 + \ldots$ to ∞, $y = 1 + b + b^2 + \ldots$ to ∞ and $z = 1 + c + c^2 + \ldots$ to ∞, then x, y, z are in A.P.

D. Both B and C

SPACE FOR ROUGH WORK